Indian Statistical Institute

B. Math. I Year

Semestral Examination 2010-2011 (Back paper)

Marks: 100 Algebra III Instructor: NSN Sastry

Answer all questions. Your answers should be complete and to the point.

1. Define elementary symmetric functions in variables over a field K. Show that they are algebraically independent over K. [10]

2. Let $R_n = M_n(K)$ be the ring of $n \times n$-matrices over a field K. Show that K^n is an irreducible R_n-module with the usual module structure.

   If T_n is the subring of R_n consisting of upper triangular $n \times n$-matrices over K, then show that K^n is not an irreducible T_n-module. $[6 + 6 = 12]$

3. (a) Define the following concepts: Noetherian ring, Artinian ring, unique factorization domain, Local ring. $[3 + 3 + 3 + 3 = 12]$

   (b) With justification, give an example for each of the following:

   (i) An Artinian ring which is not Noetherian;

   (ii) A Noetherian ring which is not Artinian;

   (iii) A local ring which is not a field;

   (iv) A commutative ring without multiplicative identity.

   $[4 + 4 + 4 + 4 = 16]$

4. (a) For a ring R, define a free R-module of rank n.

   (b) Show that any two free R-modules of rank n are isomorphic.

   (c) Which of the modules M are free R-modules:
   (i) $M = \mathbb{Q}$ and $R = \mathbb{Z}$;
   (ii) $M = K[X_1, X_2]$ and $R = K[X_1]$, K a field. $[5 + 8 + 7 + 7 = 27]$

5. (a) Construct a field of order 27.

   (b) Show that there is no field of order 12.

   (c) Does there exist a nonabelian group of order 24? How many abelian groups of order 24 exist. Justify your answer. $[8 + 7 + 8 = 23]$